### Main Ideas/Questions | Notes/Examples
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LINEAR PROGRAMMING | A method which uses linear equations + inequalities to determine the optimal solution of a certain situation.

### Finding the Optimal Solution
1. **Use the CONSTRAINTS to write a SYSTEM OF INEQUALITIES.** Constraints are the limitations on a problem.
2. **GRAPH** the system, clearly showing the FEASIBLE REGION.
3. **Write an OBJECTIVE FUNCTION.** This is the quantity you would like to minimize or maximize.
4. **TEST THE VERTICES** of the feasible region to find the OPTIMAL SOLUTION.

**Example 1:** The smallest plane in the Coast2Coast Airlines fleet offers first class and tourist class seats, and can hold a maximum of 50 passengers. In order to charter a flight, the airline must sell at least 8 first class tickets and at least 20 tourist class tickets. The company makes $40 profit for each first class ticket and $30 profit for each tourist class ticket sold. In order for the company to maximize its profits on a single flight, how many first class and tourist class tickets should they sell?

**Constraints:**
- \( x = \text{first class} \)
- \( y = \text{tourist class} \)

\[
x + y \leq 50 \quad \Rightarrow \quad y \leq -x + 50
\]
- \( x \geq 8 \)
- \( y \geq 20 \)

**Graph:**

**Objective Function:** \( P = 40x + 30y \)

**Test Vertices:**
- \((8, 20)\)
- \((3, 42)\)
- \((30, 20)\)

**Solution:** Profits are maximized by selling 30 first class tickets and 20 tourist class tickets. 

\[
P = 40(30) + 30(20) = 1800
\]
Example 2: Natalie is a hair stylist. When making appointments for her clients, she allot 30 minutes for each cut and two hours for each cut and color. She has ten hours available for appointments each day and prefers to see no more than 8 total clients. Each routine cut costs $40 and each cut and color costs $100. How many of each type of appointment should she schedule each day to maximize her income?

**Constraints:**

\[
\begin{align*}
X &= \text{Cut} \\
Y &= \text{Cut + Color} \\
\frac{1}{2} X + 2Y &\leq 10 \quad \rightarrow \quad Y \leq -\frac{1}{4} X + 5 \\
X + Y &\leq 8 \quad \rightarrow \quad Y \leq -X + 8 \\
X &\geq 0 \\
Y &\geq 0
\end{align*}
\]

**Graph:**

![Graph showing constraints]

**Objective Function:**

\[P = 40X + 100Y\]

**Test Vertices:**

\[
\begin{array}{c|c|c|c|c}
X & Y & \text{Value} & \text{Constraints} \\
1.5 & 5 & 500 & (4, 4)
\end{array}
\]

**Solution:**

Natalie will maximize her profit by doing 4 cuts + 4 cut/color.

Example 3: Baking a tray of chocolate chip cookies takes three cups of milk and two cups of flour. Baking a tray of oatmeal raisin cookies takes one cup of milk and three cups of flour. The baker has 16 cups of milk and 16 cups of flour available. He makes $2.50 profit per tray of chocolate chip cookies and $2 profit per tray of oatmeal raisin. How many trays of each type of cookies should he make to maximize his profit?

\[
\begin{align*}
X &= \text{Choc. Chip} \\
Y &= \text{Oat. Raisin} \\
3X + Y &\leq 9 \quad \rightarrow \quad Y \leq -3X + 9 \\
2X + 4Y &\leq 16 \quad \rightarrow \quad Y \leq -\frac{1}{2} X + 4 \\
X &\geq 0 \\
Y &\geq 0
\end{align*}
\]

**Objective Function:**

\[P = 2.5X + 2Y\]

**Test Vertices:**

\[
\begin{array}{c|c|c|c|c}
X & Y & \text{Value} & \text{Constraints} \\
0.4 & 3 & 8.00 & (2, 3)
\end{array}
\]

**Solution:**

He should make 2 trays of choc chip + 3 trays of oat. raisin.